## Name:

## Academic Scholarship 2011

Preliminary Examination

## MATHEMATICS

Time Allowed: $11 / 2$ hours

## No calculating aids may be used.

There are two sections on this paper.
Section A ( 50 marks) consists of shorter answers and you should spend approximately 40 minutes on this section.

Section B (70 marks) consists of longer answer questions and you should spend approximately 50 minutes on this section.

You should write your answers on lined paper and show as much working as possible. Isolated answers will gain little credit.

You should avoid spending too long on a single question.
If you are having difficulty with a particular question you should move on and return to it at the end. You should not be concerned if you cannot answer all of the questions.

At the end of the exam, both the question paper and your answers on lined paper should be handed in, with your name clearly indicated on all sheets.

## Section A:

1. Calculate
(a) $8.46+72.3 \times 4.9$
(b) $\sqrt{6.4 \times 10^{9}}$
2. Evaluate the following writing your answers as fractions in their lowest terms:
(a) $\frac{5}{12}-\frac{2}{7}$
(b) $1 \frac{4}{7} \times \frac{2}{11}$
(c) $2 \frac{1}{5} \div 2 \frac{1}{10}$
(d) $\frac{1}{2+\frac{4}{3+\frac{1}{3}}}$
3. Solve the following equations to find the value of $x$ :
(a) $\frac{x}{3}-32=51$
(b) $4(x+1)-3(x-4)=7(x-2)$
(c) $\frac{1-x}{3}-\frac{x-3}{2}=6$
4. Find the value of the expressions below when $=\frac{1}{9}$, then state which is the smallest and which is the largest

$$
\begin{array}{lllll}
\frac{1}{x} & \frac{x}{2} & x^{2} & x+1 & \frac{2}{\sqrt{x}} \tag{3}
\end{array}
$$

5. Substitute $a=-2, \quad b=-3, \quad c=5, d=\frac{1}{3}$ into the following expressions and work them out:
(a) $b(a+c+d)$
(b) $b d-a c$
(c) $b^{4}$
(d) $a^{2}-b$
6. Find the next two terms in the following sequences.
(a) $9,3,-3,-9, \ldots \ldots \ldots$
(b) $3,12,27,48, \ldots \ldots \ldots$
(c) $0.05,0.0125,0.003125$, $\qquad$
7. The price of a leather sofa was increased by $20 \%$ and subsequently decreased by a third. It ended up costing $£ 180$ less than it had done originally. By calling the original price $x$ and forming an equation, find the price it was originally.
8. In the triangle shown, the angles marked $a$ are equal and the angles marked $b$ are equal. Calculate $c$. Show all your working with reasons.


## Section B:

1. Work out by the simplest method possible :
(Complicated multiplications are NOT required and will gain NO credit)
a) $123456 \times 999999$
b) $\frac{36 \times 49 \times 54 \times 75 \times 32}{21 \times 45 \times 63 \times 24 \times 40}$
c) $123456^{2}-123456 \times 123446$
2. (i) An electrician charges a call-out fee of $£ 50$ when makes a visit for an electrical problem. He then charges an additional fee of $£ 36$ per hour of work.
(a) If he works for $t$ hours, how much will he charge? (leave $t$ in your answer)
(b) The electrician charges $£ 170$ for installing some outside lights. Write down an equation and then solve it to find out how long the job took. Give your answer in hours and minutes.
(ii) The Best Bakery sells triangular cakes at 39p and square buns 23p each. For her party, Helen buys a total of 40 of these cakes and buns and spends $£ 13.52$.
How many of each did she buy?
3. For any whole number $x$, let $R(x)$ be the remainder when $x$ is divided by 5. So, for example, $R(18)=3$ and $R(25)=0$
a) Find $R(8)$
b) Find $R(12)$
c) Work out $R(R(8) \times R(12))$
d) Work out $R(8 \times 12)$
e) Write down all the values $R(x)$ can take.

In fact, $R(a \times b)=R(R(a) \times R(b))$ for all whole numbers $a$ and $b$.
(f) Write down all the values $R\left(a^{2}\right)$ can take when $a$ is a whole number.
(g) Can a number which has 8 as its last digit be the square of a whole number? Explain your reasoning
4. The flight from London to Cape Town is 9600 km and takes 10 hours.
(a) Express this speed in metres per hour in standard form.
(b) At this speed how many centimetres could you travel in 0.02 minutes?
(c) How long, in minutes and seconds, would it take to travel 100 km ?
5. A fierce guard dog is tethered by a 15 metre chain to a post that is 9 metres from a straight path. For what distance along the path is the trespasser in danger from the dog? (Hint: draw a diagram)
6. The code for a safe is a 14 -digit number which includes two 7 s separated by seven digits, two 6 s separated by six digits, two 5 s separated by five digits, and so on all the way down to two 1 s with a single digit between them. If the first four digits of the code are 1517, crack the code.
7. The diagram shows two equal circles, both of radius 2 cm , inside a semicircle. Find the shaded area. You can leave $\pi$ in your answer.

8. In my sock drawer I have 30 socks comprised of 3 pairs of blue socks, 7 pairs of white socks and 5 pairs of red socks. Unfortunately the pairs are all separated so the individual socks are all mixed up.

If I selected at random without looking, how many socks would I need to remove from the drawer in order to be sure of having
(a) a matching pair of socks
(b) a matching pair of red socks
(c) two pairs of matching socks in different colours
9. Using exactly five 5 s and the symbols,,$+- \times, \div$, fractions and brackets it is possible to make whole numbers.
For example, number 1 can be formed using $(5-5) \times 5+\frac{5}{5}=1$
Show clearly how you can make the following numbers using exactly five 5 s.
(a) 2
(b) 8
(c) 9
10. Mrs G, Mr A and Miss T spend the evening gambling at home. At the start of the evening the amount of money they have between them is in the ratio $5: 4: 3$ respectively, while at the end of the evening the amount of money is in the ratio $4: 3: 2$ respectively. One of them loses $£ 10$. How much money did each have at the start of the evening?

